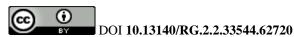
An Equivalent Beam Model for the Dynamic Analysis to a Feeding Crane of a Tall Chimney. Application in a Coal Power Plant

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ABSTRACT. The paper presents a dynamic analysis for a special crane, which serves a coal power plant. The steel cables for the lifting mechanisms of crane are long and flexible. For this reason, when is feeding the tall chimney, its can appear dangerous dynamic effects due to the suspended load. This load can perform oscillations or vibration movements. As a result, the suspended load position is sometimes difficult to control. Through experimental researches, using a special fitting with strain gauges and accelerometers assembled along the crane's arm as a beam, we have obtained relevant information. Using the initial design data, we were able to develop an optimal nonlinear dynamic model. This one was the experimental support for other simulations in extremely dangerous situations, like: the accidental fall of the suspended load from the crane hook or a mechanical strong shock due to the collision between the suspended load and the tall chimney wall or the power plant wall, under the strong wind conditions, for example.

Introduction. A crane is a type of machine, equipped in the main with a hoist rope, wire ropes or chains and pulleys, which can be used both to lift the lower materials but also to move them horizontally. It is mainly used for lifting heavy loads and transporting them to other places. Generally, by their construction, the cranes are strong mechanical structures, which must allow loads handling, regardless of the external environmental conditions. A crane model is shown in Figure 1. Graziano F. and Michel G. [1] studied more applications for the cranes under loads moving, Maczynski A. and Wojciech S. [2] have stabilized the load's position to offshore cranes and Zhou Y. and Chen S. [3] have investigated the cables breakage, but very important is that the cranes should provide the elimination of injury risk, even if this one results from abnormal predictable situations. The cranes and their lifting accessories are subject to random dynamic loads, whose emergence, size and direction of movement are very difficult to control. According to Cioara T.Gh. et al [4] and Gabbal R.D. and Simin E. [5], speed, intensity and the wind direction or the malfunction of lifting loads systems (as vibratory shock load), represent causes, which may endanger the strength crane structure. The study of percussive systems movements with a linear or nonlinear elastic characteristic was the subject of the dynamic models with more freedom degrees, which were developed by Awrejcewicz [6] and Kwon D.K. et al [7].

Getter D.J. et al [8] showed that the wind maybe causes certain dynamic and complex loads relative to some structures located in free air conditions, as: buildings, industrial installations, communications antennas and many others. These structures must to be designed so that to withstand at wind loads corresponding to the maximum intensity of these areas, as a strong gale (mean speed 22-25 m/s) or a storm (30-35 m/s), e.g. For the design, control and verification of the cranes working in such conditions, the usual methods for the vibrations study are no longer applicable.

Being equivalence with a mechanical structure having several degrees of freedom, every crane can be modeled by associating with *a function of unilateral connections* which was formulated by Paraskevopoulos E. and Natsiavas S. [9]. The function of unilateral connections is the analytical form for representation the degrees of freedom and allows the study all possible cases of movements for the mechanical structures requested to variable loads. Mainly, the function of unilateral connections allows the analytical transposition of strikes (percussions), so that to be applied the general methods of analytical mechanics, like Lagrange' equations or the conservation of energy, e.g. In this way, studying the dynamic behavior of crane in the strong wind conditions or under vibrational shock loads of the load lifting system, Harris C.M. and Piersol A.G. [10] and Silva C.W. [11] have imposed the need for determination of own frequencies to eliminate the resonance frequencies and to ensure the stability in operation.

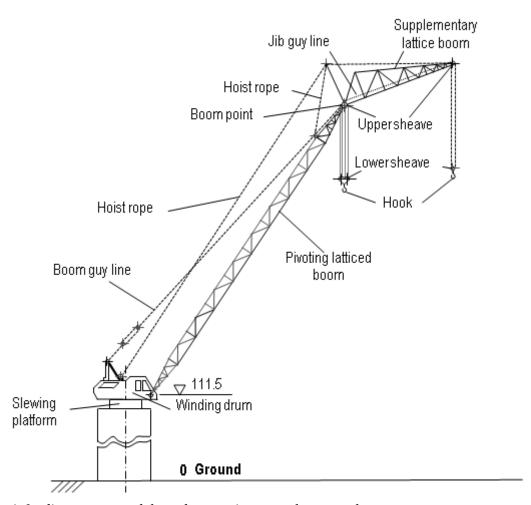


Figure 1. A feeding crane model used to serving a coal power plant.

Modelling of the testing plant.

The modelling and investigation of the feed crane was conducted in normal operating conditions. An overview of the crane that feeds the tall chimney of a coal power plant is shown in Fig. 2.

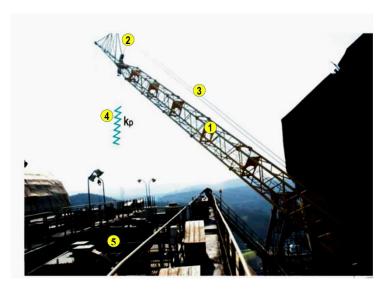


Figure 2. Overview of a crane, serving the tall chimney of a coal power plant.

Into detailed shape, a bottom view of the loading crane is illustrated in Figure 3.

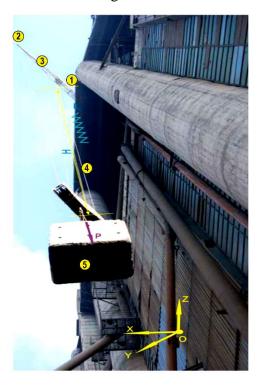


Figure 3. Bottom view of crane, loaded with a load P.

For a correct application of the unilateral connections function, we have attached to the mathematical model a coordinate system conventional established, 0XYZ. From constructively point of view, the crane is composed of a pivoting latticed boom 1 which is located at approx. 111.5 m above the ground, on the coal power plant platform serving a tall chimney (see Figure 2 and 3). The pivoting latticed boom 1 can be considered as a special structure because the suspended load 5 is lifted from ground to the supply platform situated about 120 m, without to be guided. For local manipulations of low loads, is used another supplementary pivoting lattice boom 2. The lifting of load 5, it is achieved with the help of two pulley tackle 3 (upper sheave) and 4 (lower sheave).

The load space is limited by the power plant wall and body of the tall chimney. In a possible oscillation movement of suspended load during lifting, it may come in contact with the walls of neighboring constructions. For this reason, Lukasz D. [12] showed that the position of the suspended load should be kept permanent under control.

In view to get information on modal components tensioned in the working time of crane, we have used two sensors - see Figure 4 -. A strain gauge TI it was applied on one of the four longerons of the pivoting latticed boom 1, in the middle distance L/2. This one is placed in a Wheatstone bridge circuit (the strain gauge itself is one of four resistances), and it measures elastic deformations of the pivoting jib in a critical section S_u . We used the DY1x type as strain gauge, with the 2 parallel measuring grids for measurements on bending beams.

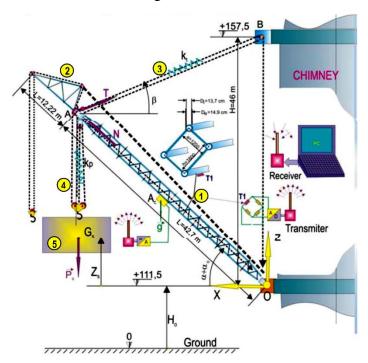


Figure 4. Schematic diagram of the crane, serving the tall chimney of a coal power plant.

To the same distance on longeron we placed an accelerometer A_c having its sensitivity axis to normal on longitudinal axis of the lattice boom. Like accelerometer we used the ADXL103 model. ADXL103 is high precision, low power, dual axis, with signal conditioned of voltage outputs, in a monolithic structure. The measuring range started from the nil frequency, so that the accelerometer was able to measure the dynamic component of static or quasi-static inclination. Both signals, from strain gauge and the accelerometer, they were amplified, modulated, converted and then wireless transmitted to a receiver, which was connected to a computer.

The constructive modelling.

According to the equations of the rotating motion of a composite beam, Georgiadis F. et al [13] demonstrated that for a discrete approximation of dynamic model, the pivoting latticed boom 1 can be considered as an equivalent elastic bending beam. This one - see Figure 5 - it hinged under the angle α_0 of the pivoting position due to the pulley tackle 3. From the dynamic study of a special crane, Cioara T.Gh. et al [4] showed that pulley tackle have an equivalent stiffness k_t .

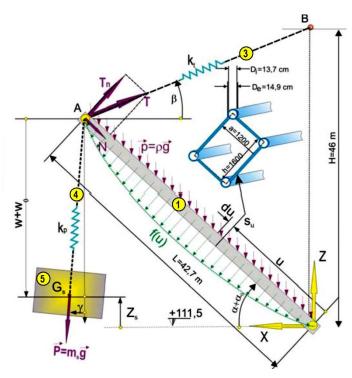


Figure 5. The scheme of the equivalent beam model of the feeding crane.

Lifting the load **5** is performed by another pulley tackle **4**, having an equivalent stiffness k_p . According to observed experimental data by Buckham B. et al [14] and Crellin E. et al [15], the instantaneous position of any section S_u , along the equivalent beam at the pivoting angle $\alpha + \alpha_0$, can be expressed in a reference system XOZ, by coordinates:

$$x_u = u \cos(\alpha + \alpha_0) - f(u) \ q(t) \sin (\alpha + \alpha_0)$$

$$z_u = u \sin(\alpha + \alpha_0) + f(u) \ q(t) \cos(\alpha + \alpha_0)$$
(1)

where u – coordinate of the section S_u in undeformed status;

f(u) – the arrow elastic deformation of the equivalent beam requested to bending according to first natural mode of the vibration motion, q(t);

 α_0 – the initial geometric position of the equivalent beam;

 α – the angular variation of the equivalent beam position under dynamic loads

The shape of the first vibration' structure mode, where the stiffness of the tall chimney and the coal power plant wall are bigger that the crane elements, it can be approximated by a static distribution. In case where the load P has a uniform distribution along the beam of length L = 42.7 m, the elastic arrow can be calculated after Bruno D. et al [16] with:

$$f_{st} = \frac{R\xi(1-2\xi^2+\xi^3)}{24},\tag{2}$$

with the notations

$$\xi = \frac{u}{L}; \qquad R = \frac{p L^4}{EI} \cos \alpha_0 \tag{3}$$

where E – a constant (Young's modulus);

I – the inertial moment into a cross section of equivalent beam

For the equivalent beam section, inertial moment is:

$$I = 4\left(\frac{h}{2}\right)^2 A; \quad A = \frac{\pi}{4}(D_e^2 - D_i^2)$$
 (4)

The mathematical modelling.

Based on the fundamentals of structural dynamic established from Craig R. and Kurdila A. [17], the mathematical modelling of crane was made by analyzing the energy balance in critical sections. Therefore:

Kinetic energy

Using the interface stresses and fracture energies by Bruno D. et al [16], kinetic energy of the equivalent beam has the integral form:

$$E_{k} = \frac{1}{2} \int_{0}^{L} \rho \left(\dot{x}_{u}^{2} + \dot{y}_{u}^{2} \right) du \tag{5}$$

where $\rho = p/g$ ($g = 9.81 \text{ m/s}^2$) is mass per unit length of the equivalent beam.

Taking into account a modal distribution along the length L of equivalent beam and the decomposition of governing the equations of nonlinear system formulated by Awrejcewicz J. et al [18], the kinetic energy expression (5) becomes:

$$E_k = \frac{1}{2} J \alpha^2 + \frac{1}{2} S_q q^2 + S_{\alpha q} \alpha q$$
 (6)

Where

$$J = \int_{0}^{L} u^{2} \rho \, du; \, S_{q} = \int_{0}^{L} \rho(f(u)^{2}) \, du; \, S_{aq} = \int_{0}^{L} \rho \, f(u) u \, du$$
 (7)

and these values can be calculated from the design data.

The second kinetic energy of dynamic system which was studied by Cires I. and Nani V.M. [19], is a load considered as a point mass in the gravity center G_s , having the coordinates:

$$x_s = L \cos \alpha - (w_0 + w) \sin \gamma$$

$$z_s = L \sin \alpha - (w_0 + w) \cos \gamma$$
(8)

so that the kinetic energy of load becomes:

$$E_k = \frac{1}{2} m_S (x_S^2 + \dot{z}_S^2) \tag{9}$$

which, by substitution (8) into (9), leads to final form:

$$E_{k} = \frac{1}{2} m_{s} (L\alpha)^{2} + \frac{1}{2} m_{s} (\dot{w})^{2} + \frac{1}{2} (w_{0} \dot{\gamma})^{2} + m_{s} L\alpha \dot{w} \cos(\alpha_{0} - \gamma) - \\ -m_{s} l\alpha w_{0} \dot{\gamma} \sin(\alpha_{0} + \gamma) + m_{s} w_{0} \dot{w} \dot{\gamma} \sin 2\gamma)$$
(10)

Strain energy

The strain energy of the dynamic system under the equivalent beam form it consists of three components:

a) The binding energy of the equivalent beam 1 under its own weight

Using the computational dynamics of an elastic string pendulum attached to a rigid body, which was formulated by Taeyoung L. et al [20], the binding energy of the equivalent beam under its own weight, has the form:

$$E_{s_b} = \frac{1}{2} k_b q^2 \tag{11}$$

Where

$$k_b = EI \int_0^L \left[\frac{\partial^2 (f(u))}{\partial u^2} \right]^2 du \tag{12}$$

is the equivalent stiffness of the equivalent beam 1.

b) The strain energy of beam due to stretching of the hoist ropes in pulley tackles 3

According to Koh C. et al [21] who studied the low-tension cables dynamics, the k_t elastic stiffness of the hoist ropes in the pulley tackle 3 is:

$$k_t = n_c \frac{E_{pt}I_{pt}}{AB} \tag{13}$$

where E_{pt} and I_{pt} are Young's modulus and the inertial moment of a single strand;

 n_c – number of stranded wire

The strain energy due to stretching of the hoist ropes in pulley tackle **3** is after Buckham B. et al [14] and Koh C. et al [21]:

$$E_{S_{pt3}} = \frac{1}{2} k_t [(AB)_t - (AB)_0]^2$$
 (14)

where: $(AB)_t$ - instantaneous distance at time t between two points, A and B; $(AB)_0$ - is same distance having the system unloaded corresponding to angular position α_0 of the equivalent beam

From the geometric conditions:

$$(AB)_0 = \sqrt{(L\cos\alpha_0)^2 + (L\sin\alpha_0 - H)^2}$$
 (15)

the instantaneous position can be considered as a small variation of the α parameter ($\alpha < 5^0$, $\sin \alpha \approx \alpha$, $\cos \alpha \approx I$), which is added to α_0 , resulting:

$$(AB)_t - (AB)_0 = R_\alpha \alpha \tag{16}$$

where:

$$R_{\alpha} = \frac{-LH\cos\alpha_0}{\sqrt{(L\cos\alpha_0)^2 + (L\sin\alpha_0 - H)^2}}$$
(17)

Finally, from (14), we obtained

$$E_{s_{pt3}} = \frac{1}{2} k_t R_\alpha^2 \alpha^2 \tag{18}$$

c) The strain energy of beam due to stretching of the hoist ropes in pulley tackle 4

This energy is resulting from the load lifting motion variation. The elastic k_p stiffness of the hoist ropes in the pulley tackle **4** is according to Buckham B. et al [14] and Koh C. et al [21]:

$$k_{p} = n_{c} \frac{E_{pt} I_{pt}}{w_{0}} \tag{19}$$

where: E_{pt} and I_{pt} are Young's modulus and the inertial moment of a single strand; n_c – number of stranded wire; w_0 - initial position of load 5

Thus, like section b), the strain energy due to stretching o the hoist ropes in the pulley tackle 4 is

$$E_{S_{pt5}} = \frac{1}{2} k_p w^2 \tag{20}$$

Potential energy

Because we studied the movements in a vertical plane, for the energetic balancing of mechanical ensemble, we must take into account from the potential energy. After the applications of the beams under moving loads described in Graziano F. and Michel G. [1], the potential energy for the beam deformation, it has the form:

$$E_{p_b} = g \, m_b \, \frac{L}{2} \sin \left(\alpha + \alpha_0 \right) + \rho \, g \, q \cos \left(\alpha + \alpha_0 \right) F \; ; \quad F = \int_0^L f(u) \, du \tag{21}$$

and for any given load, we can calculate:

$$E_{p_1} = m_S g z_S = m_S g \left[L \sin(\alpha + \alpha_0) - (w + w_0)(1 - \cos \gamma) \right]$$
 (22)

The calculation formulas of all energies developed above were used to build a system of differential equations governing the feeding crane movements, which are defined by the column vector developed by Georgiadis F. et al [13]:

$$\{\Psi(t)\} = \{\alpha(t) \quad q(t) \quad w(t) \quad \lambda(t)\}^T \tag{23}$$

and using Lagrange's relationship:

$$\frac{d}{dt} \left(\frac{\partial E_c}{\partial \{\Psi\}^T} \right) - \frac{\partial E_k}{\partial \{\Psi\}^T} + \frac{\partial E_s}{\partial \{\Psi\}^T} + \frac{\partial E_p}{\partial \{\Psi\}^T} = \{0\}$$
 (24)

we obtained the general equation of free vibrations undamped for the entire mechanical structure:

$$[M]\{\ddot{\Psi}\} + [K]\{\Psi\} = \{0\}$$
 (25)

In assumption that the tall chimney stiffness and the coal power plant wall stiffness are too bigger compared to the crane elements, then [M] and [K] are:

$$[M] = \begin{bmatrix} J + m_s L^2 & S_{\alpha q} & 0 & -m_s L w_0 \sin \alpha_0 \\ S_{\alpha q} & 0 & 0 & 0 \\ 0 & 0 & m_s & 0 \\ -m_s L w_0 \sin \alpha_0 & 0 & 0 & m_s \end{bmatrix}$$
(26a)

$$[K] = \begin{bmatrix} k_t R_{\alpha}^2 - g m_b \frac{L}{2} \sin \alpha_0 + g m_s L \cos \alpha_0 & 0 & 0 & 0 \\ 0 & k_b & 0 & 0 \\ 0 & 0 & k_b & 0 \\ 0 & 0 & 0 & m_s g w_0 \end{bmatrix}$$
 (26b)

where [M] – is equivalent mass;

[K] – is equivalent stiffness of the crane;

 $\{\Psi\}$ – is a random conventional variable and represents the function of unilateral connections

Simulation and experimental results.

Experimental simulation was performed under actual conditions operating of the feeding crane. It has been placed into a dangerous area. When the tall chimney of the coal power plant is feeding can appear more rocking situations of load during the load lifting. The load can touch such the power plant walls or the tall chimney construction. In this way, the danger of an accident is imminent.

At various heights, we intentionally simulated few oscillations of the lifting load, in a plane parallel to the walls of adjacent buildings. The amplitude of oscillation it was high enough to enter into the dangerous area operating.

Experimentally, it was raised a load of 6000 kg. The load was subjected to forced oscillations in 5 different areas on the lifting height: 5, 30, 60, 90 and 110 m above the ground. In each area, the two sensors (strain gauge and the accelerometer) have provided on-line information regarding to the vibrational amplitude variation and stresses induced into the pivoting latticed boom 1 of the feeding crane, namely in S_u critical section. The relative position of the crane boom compared to the plane XOZ it was determined using a MTS360-232 clinometer.

Results were considered relevant to a loading height of 5 m above the ground. To this height, the oscillation motion was sufficient to determine energies, frequency and vibration amplitude which led to structural changes in the critical section.

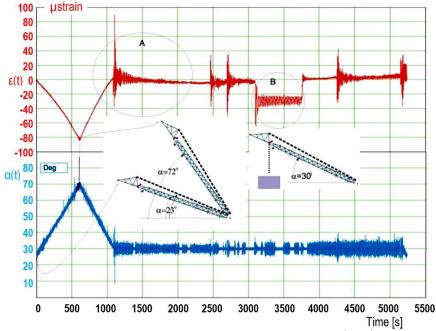


Figure 6. The time variation of stresses and the angle position $\alpha = 23^{\circ}-72^{\circ}$.

In Figure 6 are presented the diagrams of the two signals. In the first phase, it is observed a slow form of tilt to the angular position $\alpha = 23^{0}$ to 70^{0} during of 600 s, without that the load m_{s} to be hanged. In these conditions, close to a static functioning, the stresses T into hoist ropes of the pulley tackle 4 from the pivoting latticed boom 1, are:

$$T = \rho g \frac{L}{2} \frac{L \cos \alpha_0}{\sin (\alpha_0 + \beta)}$$
 (27)

The two components of the tension T are: the normal tension T_n , which is perpendicular to the beam axis, and also the axial tension N:

$$T_n = T\sin(\alpha_0 + \beta); \quad N = T\cos(\alpha_0 + \beta)$$
 (28)

The bending moment in the middle section of the beam, at L/2 is:

$$M_i = T_n \frac{L}{2} - \rho g \frac{L^2}{4} \cos \alpha_0 \tag{29}$$

and the axial force in the same section has the form:

$$N_m = N + \rho g \frac{L}{2} \sin \alpha_0 \tag{30}$$

Strain ε , at the location where sensors were positioned on the longeron, can be calculated using formula developed by Crocker [22]:

$$\varepsilon = \frac{1}{E} \left(\frac{M_i h}{2I} - \frac{N_m}{4A} \right)$$

which is in accordance with experimental researches, the maximum compression strain being attained at -80 μ strain (μ Pa), relative to position α_0 of the beam.

The dynamic effect was more intense at the test end, when the rocking movement of the beam it was stopped to angular position $\alpha_0 \approx 30^0$ (detail A of Figure 6). At that time, we recorded a transient dynamic strain by magnitude 40 µstrain (graph a), Figure 7). This one contain a modal component frequency at f = 0.12378 Hz (graph b), Figure 7), which was confirm in Nastac S. and Leopa A [23].

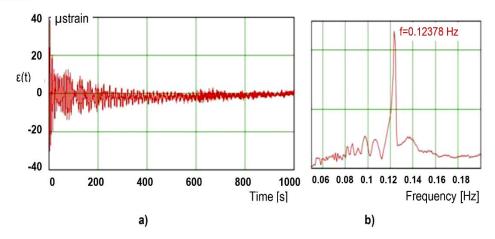


Figure 7. Time history a), and its spectrum b), for strain during a transient break in a particular position of the pivoting latticed boom.

In detail B of Figure 6 and in the next chart of Figure 8 are presented the diagrams of time history for dynamic strain and its spectrum, which were recorded during lifting the load having the mass of $m_s = 6\,000\,\mathrm{kg}$.

The value of transient dynamic strain is approximately 60 µstrain (graph a), Figure 8). From the time history diagrams and the spectrum's strain, results two peaks, one which corresponds to frequency f= 0.04503 Hz (graph b), Figure 8), and the other having same frequency of 0.12378 Hz as in spectrum b) (graph b), Figure 7). Measured value 0.04503 Hz it corresponds to the loads of a pendulum hinged by a long cable $w_0 = 110-120$ m, resulting from

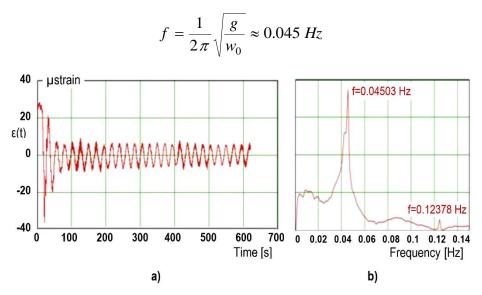


Figure 8. Time history a), and its spectrum b) of a strain during lifting the load.

The second measured value of frequency 0.12378 Hz it corresponds to a rocking denoted α of the vibration motion; this frequency component belong to both situations, when the rocking movement of the beam is stopped at angular position $\alpha_0 \approx 30^0$ during lifting the load, having a mass $m_s = 6\,000$ kg.

Discussions.

The simulation experiment confirmed the mathematical model presented at point 3 (see relationships 24 and 25), based on the function of unilateral connections and conservation of energy applied on the equivalent beam. Results obtained have provided sufficient reliable data for adopting some constructive solutions to limit and avoid the mechanical stresses close to critical values.

In extremely dangerous situations, automatically crane it stops and it is locks lifting the load, eliminating the human operator intervention. If the dynamic structure of the crane not it stabilizes, then all automatically the load is lowered urgently on the ground.

For this reason, in such industrial applications, is sometimes necessary the adoption of a special construction for the feeding crane safety.

Summary. From the experimental research made in normal operating conditions of the feeding crane, the following conclusions can be drawn:

- Dynamic effect is more intensive at the end of operation, when the rocking motion of the beam is stopped at angular position $\alpha_0 \approx 30^0$ and during lifting the load having a mass $m_s = 6\,000\,kg$.
- During the transient dynamic period, can appear a variable strain having the magnitude 40-80 µstrain, which is sometimes dangerous for the mechanical structure of crane.
- From graph time, results two peaks of low frequency:
- the frequency f=0.04503 Hz correspond to the oscillating load, where the load is hinged by a long cable; in this case, the crane operator should avoid the starting/stopping with shocks;
- the frequency f=0.12378 Hz correspond to the pendulum motion, so the crane operator should avoid the crane's working in strong winds.

An actual and important objective for designers is to design a new pivoting jib crane as a lattice beam in a lighter structure. From the experimental simulation data, we found that there is the possibility to adopt a flexible constructive structure for the same nominal load lifted.

This paper presents only a portion of experimental undertaken researches, which can continues in the future. The results obtained are useful for designer to optimizing the crane conception in a light structure, flexible and resistant to variable and random loads.

This objective refers to improving the performances of crane, in particular to the operational reliability at high speeds by operating.

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